

# BENDING STRESS

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## Chapter 6



The deck of this bridge has been designed on the basis of its ability to resist bending stress.

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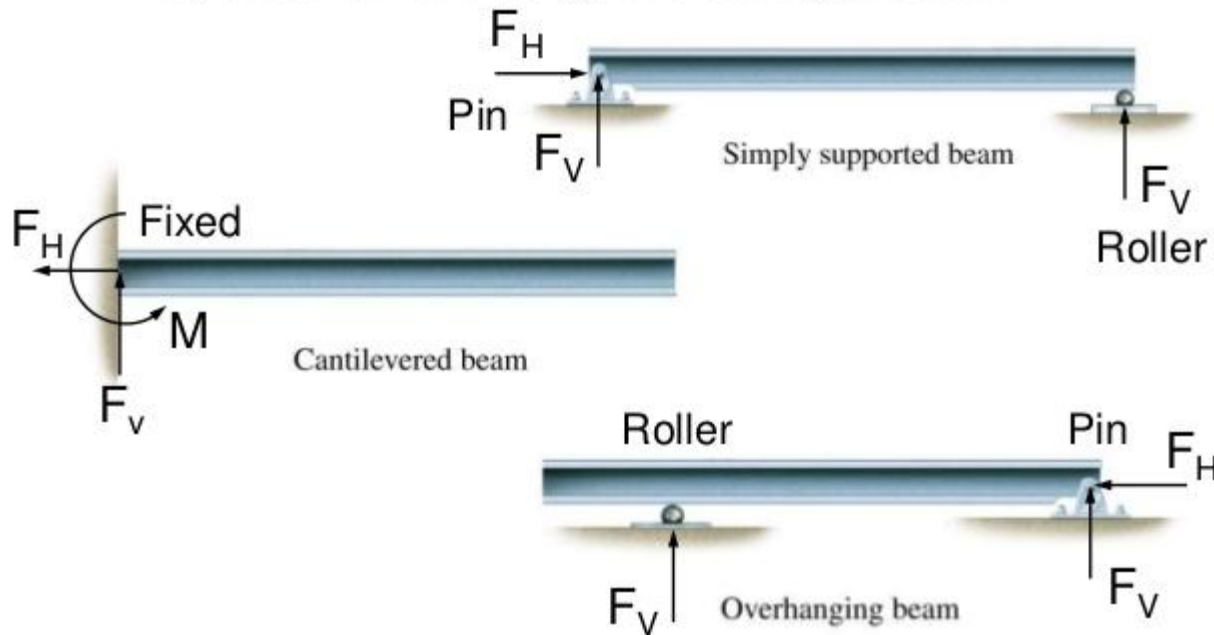
## 6.1 Shear and Moment Diagrams

- Members that are slender and support loadings that are applied perpendicular to their longitudinal axis are called *beams*.
- In general, beams are long, straight bars having a constant cross-sectional area. Often they are classified as to how they are supported.



## Types of Beams

- Depends on the support configuration

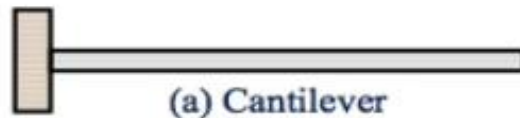


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## Beam Types

❖ Types of beams- depending on how they are supported.



(a) Cantilever



(b) Simply supported



(c) Overhanging



(d) continuous



(e) Fixed ended



(f) Cantilever, simply supported



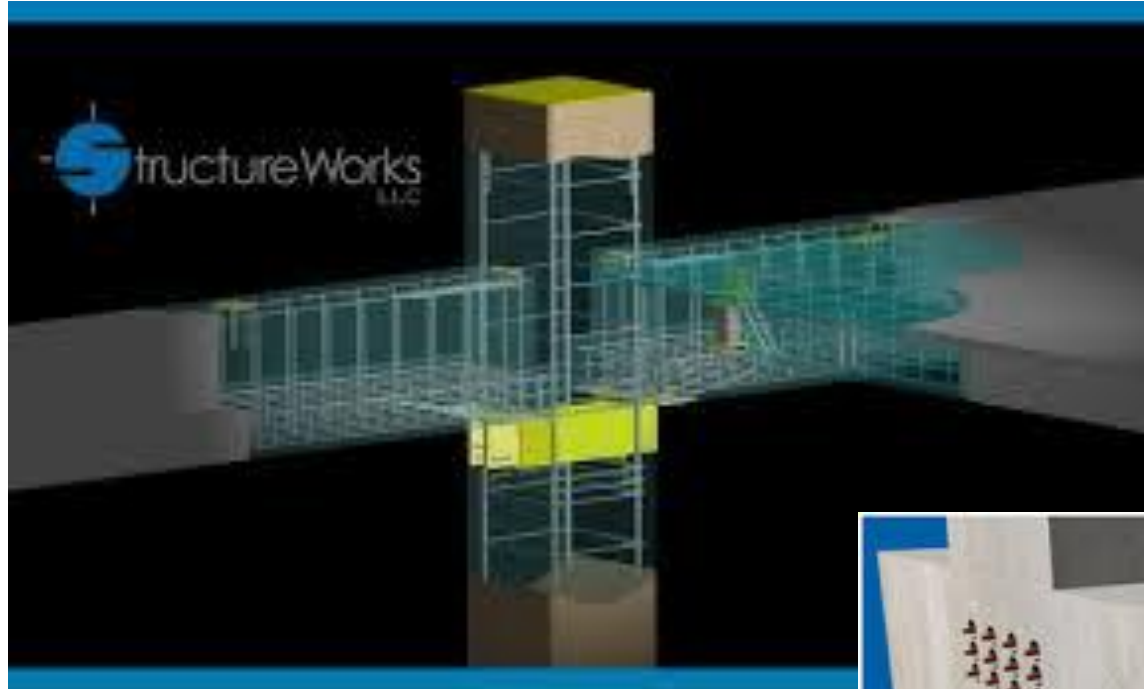
## 6.1 Shear and Moment Diagrams

- In order to properly design a beam it therefore becomes necessary to determine the *maximum* shear and moment in the beam.
- One way to do this is to express  $V$  and  $M$  as functions of their arbitrary position  $x$  along the beam's axis.
- These *shear and moment functions* can then be plotted and represented by graphs called ***shear and moment diagrams***.
- The maximum values of  $V$  and  $M$  can then be obtained from these graphs.
- The shear and moment diagrams provide detailed information about the *variation* of the shear and moment along the beam's axis,
- They are often used by engineers to decide where to place reinforcement materials within the beam or how to proportion the size of the beam at various points along its length.

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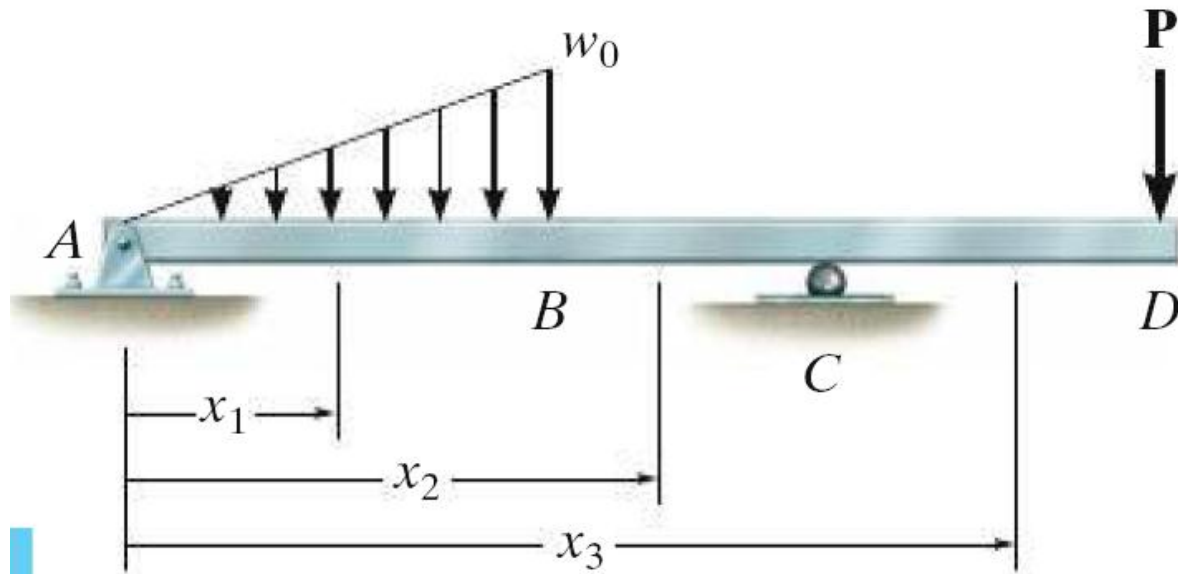
## Beam reinforcement



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Shear and bending-moment functions must be determined for each region of the beam between any two discontinuities of loading

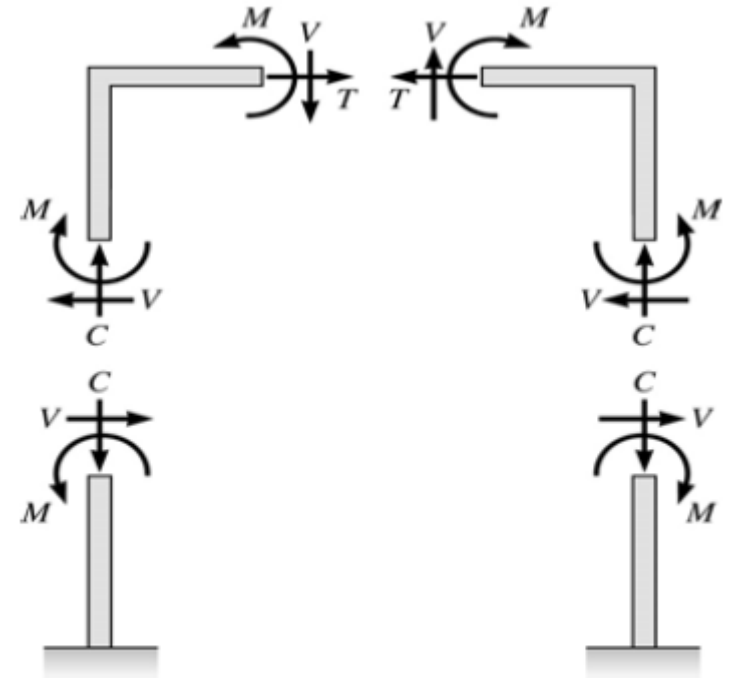
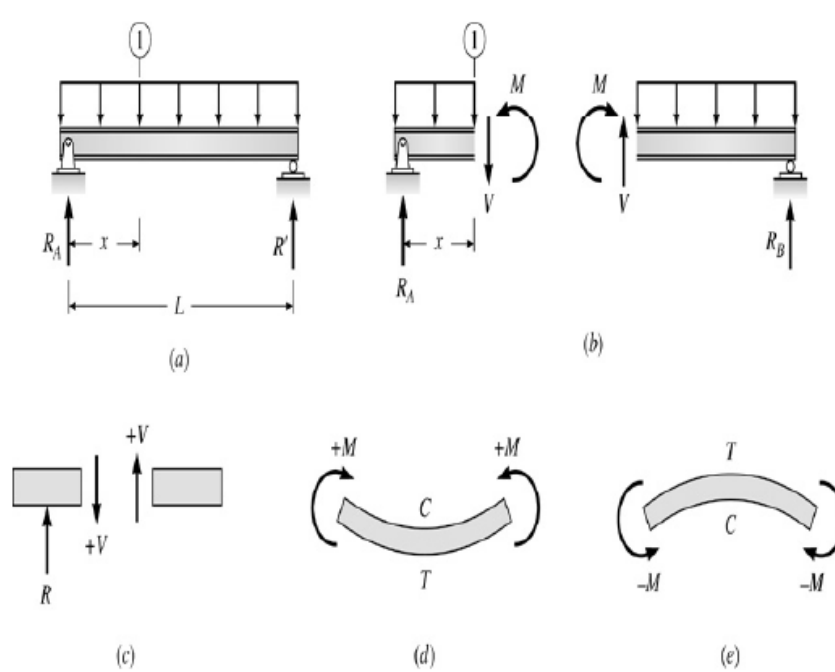




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## Beam sign Convention for Shear and Moment



Positive Internal Forces Acting on a Portal Frame

## Important Points

- *Beams* are long straight members that are subjected to loads perpendicular to their longitudinal axis. They are classified according to the way they are supported, e.g., simply supported, cantilevered, or overhanging.
- In order to properly design a beam, it is important to know the *variation* of the internal shear and moment along its axis in order to find the points where these values are a maximum.
- Using an established sign convention for positive shear and moment, the shear and moment in the beam can be determined as a function of its position  $x$  on the beam, and then these functions can be plotted to form the shear and moment diagrams.



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Ex1:- Draw the shear and moment diagrams for the beam shown in Fig. 6-4 a .

## SOLUTION

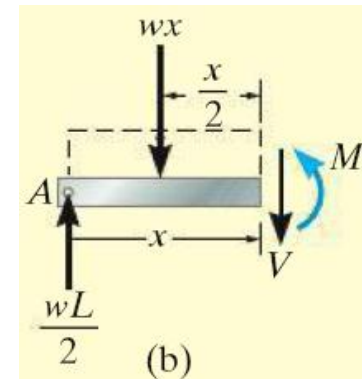
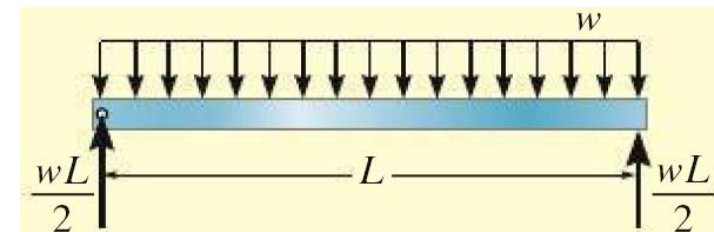
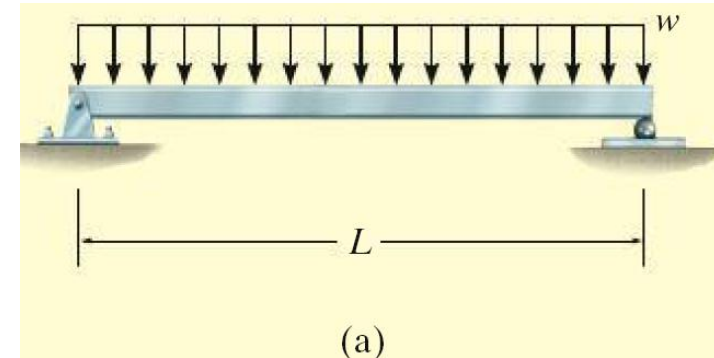
Support Reactions. The support reactions are shown in Fig. 6-4 c .

$$+\uparrow \Sigma F_y = 0; \quad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \quad (1)$$

$$\zeta + \Sigma M = 0; \quad -\left(\frac{wx}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w}{2}(Lx - x^2) \quad (2)$$



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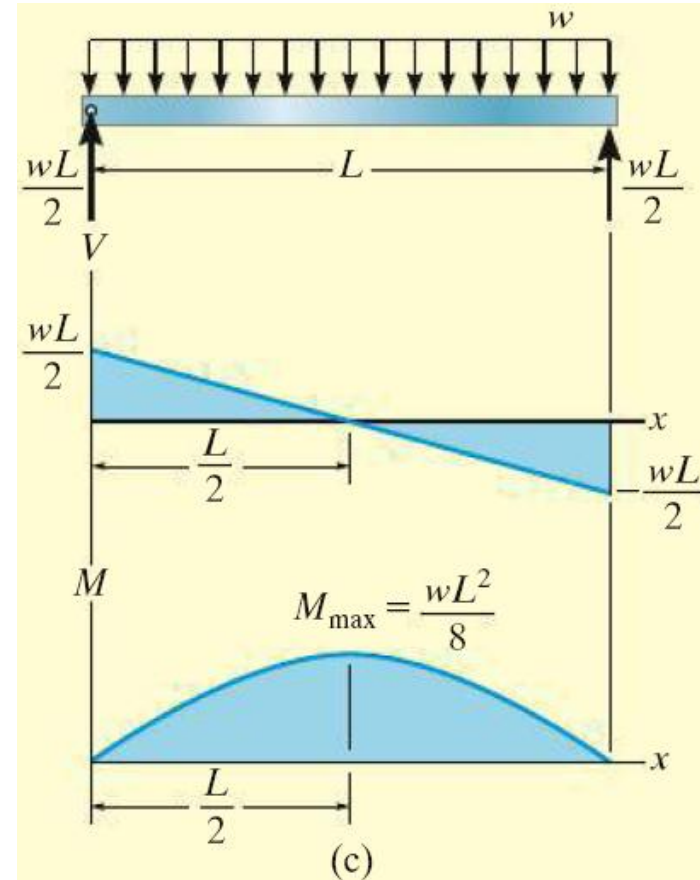
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Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 6-4 *c* are obtained by plotting Eqs. 1 and 2 . The point of *zero shear* can be found from Eq. 1 :

$$V = w\left(\frac{L}{2} - x\right) = 0$$
$$x = \frac{L}{2}$$

**NOTE:** From the moment diagram, this value of  $x$  represents the point on the beam where the *maximum moment* occurs, since by Eq. 6-2 (see Sec. 6.2 ) the *slope*  $V = dM/dx = 0$ . From Eq. 2 , we have

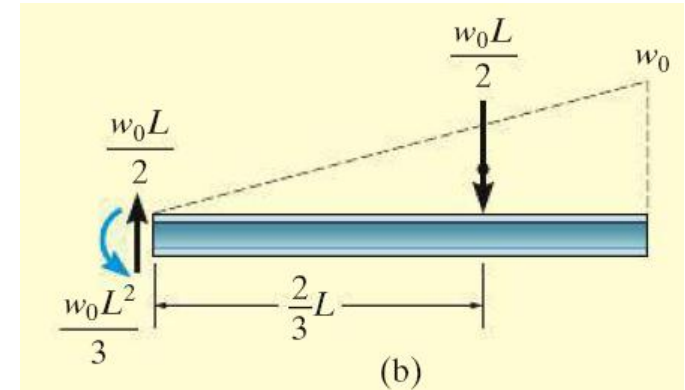
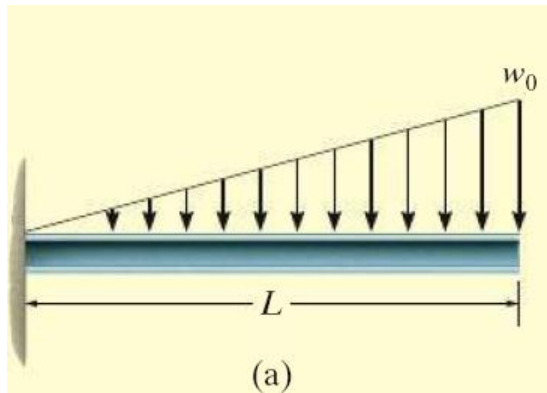
$$M_{\max} = \frac{w}{2} \left[ L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2 \right]$$
$$= \frac{wL^2}{8}$$



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Ex1:- Draw the shear and moment diagrams for the beam shown in Fig. 6-5 a .

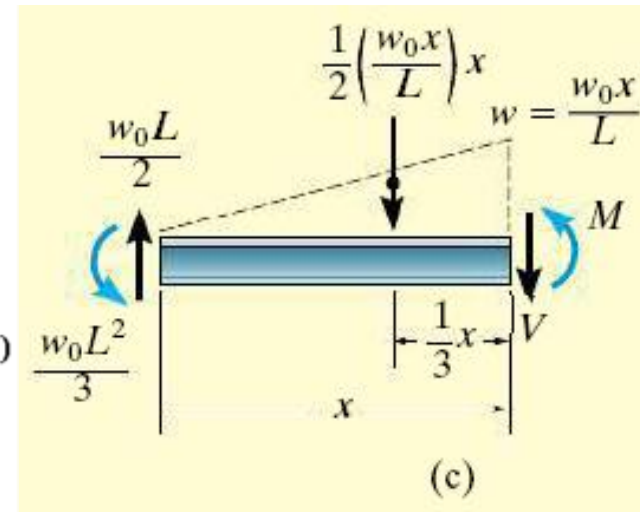


$$+\uparrow \Sigma F_y = 0; \quad \frac{w_0 L}{2} - \frac{1}{2} \left( \frac{w_0 x}{L} \right) x - V = 0$$

$$V = \frac{w_0}{2L} (L^2 - x^2) \quad (1)$$

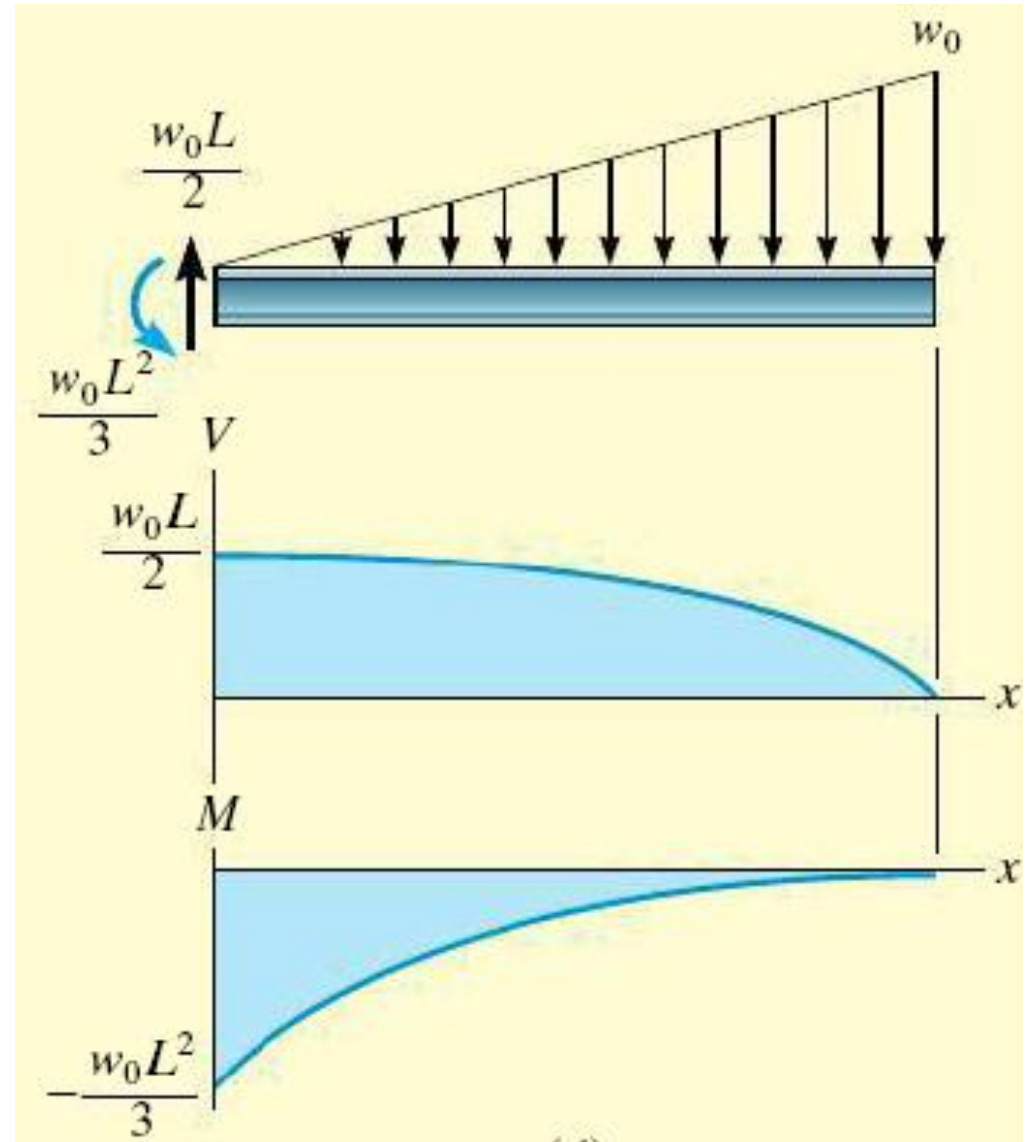
$$\zeta + \Sigma M = 0; \quad \frac{w_0 L^2}{3} - \frac{w_0 L}{2} (x) + \frac{1}{2} \left( \frac{w_0 x}{L} \right) x \left( \frac{1}{3} x \right) + M = 0$$

$$M = \frac{w_0}{6L} (-2L^3 + 3L^2 x - x^3) \quad (2)$$



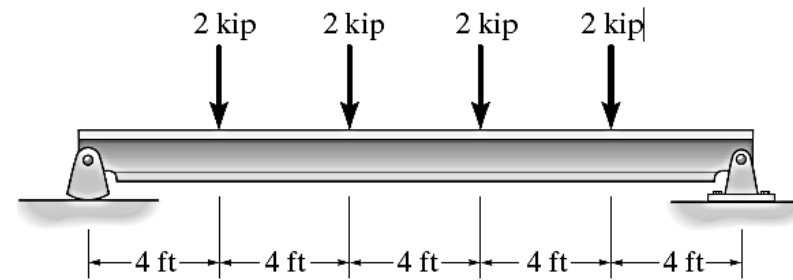
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## Homework

1. Draw the shear and moment diagrams for the beam.



2. Draw the shear and moment diagrams for the beam.

